Almost all odd numbers can be understood as the sum of three prime numbers (alternative proof)

Terminological foreword

By the expression “almost all”, referred to an infinite set of integer numbers, we’ll mean “all except a finite amount”.

Definition A

By "*Vinogradov's Theorem*" we mean the statement that can be read on the homonymous page of the English Wikipedia (*https://en.wikipedia.org/wiki/Vinogradov%27s_theorem*), from which it can be deduced that almost all odd numbers Nd can be understood as the sum of three not necessarily distinct prime numbers, i.e. that Nd = Px + Py + Pz.
Below we will find an alternative proof of the main consequence of Vinogradov's theorem, i.e. that almost all odd numbers Nd can be understood as the sum of three not necessarily distinct prime numbers, i.e. that Nd = Px + Py + Pz.

Definition B

By "*Prime number theorem*" we mean the statement according to which the ratio between the number of prime numbers between 0 and x and the function x/ ln x approaches 1, as the number x under consideration becomes always bigger.

Definition C

By "*Result of Wen Chao Lu*", we mean the article "*Exceptional set of Goldbach number*", published by Wen Chao Lu, which can be summarized in the sentence, taken from the abstract of the same article, according to which "*Let E(x) the number of even numbers not greater than x that cannot be written as the sum of two primes. In this article it is proved that* $E(x) << x^{0.879}$ ”.

Proposition 01

Almost all odd numbers Nd can be understood as Nd = Px + Py + Pz.

Proof

Take a generic odd number x: by the Prime number theorem, between 0 and x there are x/ln x prime numbers between 0 and x; in this way we obtained a list of prime numbers P', P'', P''' etc. We subtract the various P', P'', P''' from the odd number x from which we started: obviously we obtain from time to time even numbers Npa which are always different as a result of the subtraction.

x – P' = Npa'
x – P'' = Npa''
x – P''' = Npa'''
…
x – P(n) = Npa(n)

Assuming that the various even numbers Npa', Npa'', Npa''' that we have thus obtained can only be either exceptions to Goldbach's Conjecture or confirmations to Goldbach's Conjecture, and that therefore if an even number is not an exception to the Conjecture of Goldbach then this even number is a confirmation of Goldbach's Conjecture, we can calculate the number of exceptions

between 0 and x: for Wen Chao Lu's result, we can state that "*Let E(x) the number of even numbers not greater than x that cannot be written as the sum of two primes. In this article it is proved that* $E(x) << x^{0.879}$ ".
From results of mathematical analysis, it can be shown, in general, that x/ln x grows faster than any power of x with exponent less than 1; furthermore the function E(x), as stated by Wen Chao Lu, is a lower value than x raised to the first; therefore, from a certain x onwards, i.e. for almost all x, by repeating the procedure described above, one will have that at least one of the even numbers Npa', Npa'', Npa''' etc. is a confirmation of Goldbach's Conjecture , and of course

x – P' = Npa'
x – P' = Px + Py
x = Px + Py + P'
x = Px + Py + Pz

As we see, "*almost all odd numbers Nd can be understood as Nd = Px + Py + Pz"*. - CVD

Proposition 02

The writings of an odd number Nd as Nd = Px + Py + Pz tend to infinity.

Proof

On the basis of *Proposition 01*, we can state that "*almost all odd numbers Nd can be understood as Nd = Px + Py + Pz*": since the confirmations to Goldbach's Conjecture are infinite, for sufficiently large odd numbers Nd we can not only state that these numbers can be written as Nd = Px + Py + Pz, but also that these writings tend to infinity, because the reasoning set out in the *Proposition 01* must necessarily lead to infinite prime numbers which, subtracted from the number x taken into consideration , give infinite confirmations to Goldbach's Conjecture, precisely because, as we said, x/ln x grows more rapidly than any value between x raised to zero and x raised to the first.
As we see, "*the writings of an odd number Nd such as Nd = Px + Py + Pz tend to infinity*".

Final considerations

How far are we from actual Goldbach's Weak Theorem? And how much is the method proposed here "simpler" than the original results of I. M. Vinogradov and H. Helfgott? For anyone among the kind readers who can get caught up in enthusiasm (and I got caught up quite quickly), it may be useful to keep in mind how much the *Team* of *LetsproofGoldbach*! commented on the issue, in the preparatory email exchange for this text:

"*As we mentioned in a comment in your document, the number of 10 raised to the fifteenth came out only by comparing the asymptotic orders, but not the quantities to which they refer. Basically, it goes like this:*

Wen Chao Lu's Theorem implies that,for large enough numbers, so for x greater than or equal to some N, the number of exceptions between 0 and x is smaller than $x^{0.879}$.
The prime number theorem implies that, for sufficiently large numbers, therefore for x greater than or equal to a certain M, the ratio between the quantity of integers of the type P1 + 3 between 0 and x, and the function x / ln(x) , is as close to 1 as we like (let's say it's between 0.99999 and 1.00001; obviously M depends on the degree of accuracy chosen).

So simply comparing the functions $x^{0.879}$ *and x / ln(x) is only part of the story. Indeed:*

- *we don't know what N is worth (it could be much larger than 10 to the fifteenth);*
- *we don't know how much M is worth (same observation as above);*
- *we do not know how much is the difference between the quantity of numbers of type P1 + 3 between 0 and x, and the value of the function x / ln(x); the comparison between the two is based only on their relationship.*

So in principle your idea may actually be an alternative way to prove Goldbach's weak conjecture, but these important details are missing. Then, assuming we have adequately treated the three points above, N could be greater than 10 at the eighteenth; in this case the current empirical checks would not be sufficient for the remaining numbers.
This doesn't mean that on an intuitive level your speech doesn't make sense. In fact, demonstrations often come out just like this: we start from an intuition, which perhaps arrives in the blink of an eye, and then it takes weeks or months (if not more) to transform that intuition into a correct and complete.

As for the issue of simplicity, it is a moot point. Keep in mind that you have in fact used Wen Chao Lu's Theorem, which has a complexity comparable to Helfgott's! Clearly, the discussion may seem simple if Wen Chao Lu's Theorem is known, but if instead it is also necessary to study that, then everything becomes very complicated. The simplicity of a proof in general is subjective and depends on the prerequisites one assumes and in general on how deep one chooses to go; only after having agreed on these parameters can one speak objectively of the simplicity or complexity of a proof".