Definition A

Let us define as “Goldbach's Conjecture” the statement for which every integer even number (Npa) greater or equal to 4 can be intended as the sum of two prime numbers (P1; P2), not necessarily different (Npa = P1 + P2; Npa = 2 P1). It is possible that an even number Npa holds more than one representation (e. g. 14 = 7 + 7, and 14 = 11 + 3, or 16 = 11 + 5, and 16 = 13 + 3).

Definition B

Let us define as “G1 Set” the set made up of the sum of two not necessarily different prime numbers, the sum of which is an even number.

Let us define as “G2 Set” the set made up of the sum of four not necessarily different prime numbers, the sum of which is an even number (Npa = P1 + P2 + P3 + P4, where P1, P2, P3 and P4 are not necessarily different).

Let us define as “E Set” the set made up of the integer even numbers that can not be intended as the sum of two prime numbers (P1; P2), not necessarily different.

Proposition 01

Every element of the G2 Set can be intended as composed of two not necessarily different elements of the G1 Set.

Proof

By reductio ad absurdum, let us suppose that in the formula Npa = P1 + P2 + P3 + P4, the prime number P4 and only P4 is equal to 2

$$Npa = P1 + P2 + P3 + 2$$

This is false: since P1, P2 and P3 are prime numbers different from 2, (P1 + P2 + P3) is an odd number (Nd), and the sum of an even number and an odd number is an odd number, not an even number.

$$Npa = (P1 + P2 + P3) + 2$$
$$Npa = Nd + 2$$
$$Npa = Nd$$

By reductio ad absurdum, let us suppose that in the formula Npa = P1 + P2 + P3 + P4, the prime numbers P4 and P2, and only P4 and P2, are equal to 2: as a matter of fact, since P1 and P3 are prime numbers different from 2, if we apply the commutative property, it is always possible to obtain two elements of the G1 Set, because P1 and P3 are never equal to 2 and (2 + 2) is an element of the G1 Set.

$$Npa = P1 + 2 + P3 + 2$$
$$Npa = P1 + P3 + 2 + 2$$
$$Npa = (P1 + P3) + (2 + 2)$$

By reductio ad absurdum, let us suppose that in the formula Npa = P1 + P2 + P3 + P4, the prime numbers P2, P3 and P4, and only P2, P3 and P4 are equal to 2: then again, we would obtain the sum of an even number and an odd number, since P1 is a prime number always different from 2, and clearly this sum is not an even number.

$$Npa = P1 + (2 + 2 + 2)$$
$$Npa = P1 + Nd$$
$$Npa = Nd$$

Finally, let us suppose that in the formula Npa = P1 + P2 + P3 + P4, the prime numbers P1, P2, P3 and P4 are all equal to 2: we obtain twice an element of the G2 Set, since (2 + 2) is an element of the G1 Set.

$$Npa = 2 + 2 + 2 + 2$$
$$Npa = (2 + 2) + (2 + 2)$$

It is clear that in the formula Npa = P1 + P2 + P3 + P4 the required prime numbers are equal to 2 if and only if at least another prime number is equal to 2, or, alternatively, if and only if all the prime numbers required are equal to 2. This implies that every element of the G2 Set is intended as composed of the sum of two not necessarily different elements of the G1 Set. - QED

Proposition 02

The elements of the G2 Set are intended as composed of the sum of every element of the G1 Set with every element of the G1 Set, not necessarily different, or, if we prefer, the G2 Set is intended as G1 + G1.

Proof

As for Thesis 01, every element of the G2 Set is composed of elements of the G1 Set, and, since we already know that every even number grater or equal to 8 can be intended as Npa = P1 + P2 + P3 + P4, let us sum every element of the G1 Set with itself.

$$Npa = (2 + 2) + (2 + 2)$$
$$Npa = (3 + 3) + (3 + 3)$$
$$Npa = (5 + 5) + (5 + 5)$$
$$\dots$$
$$Npa = (P1 + P1) + (P1 + P1), \text{ or, if we prefer, } Npa = (2\ P1) + (2\ P1)$$

By doing so, it is not possible to obtain every even number greater or equal to 8: given that, as for Thesis 01, the elements of the G2 Set are composed of the not necessarily different sums of elements of the G1 Set, the G2 Set would have less elements than required, so it is necessary to add every sum of different elements of the G1 Set to the G2 Set, since the G1 Set, and only the G1 Set, contains the necessary elements for the G2 Set.
This implies that the elements of the G2 Set are intended as composed of the sum of each element of the G1 Set with each element of the G1 Set, not necessarily different from each other, or, if we prefer, the G2 Set is intended as G1 + G1. - QED

Proposition 03

Every even number greater or equal to 4 is equal to an element of the G1 Set or equal to the sum of two not necessarily different elements of the G1 Set, or, if we prefer, every even number greater or equal to 4 is equal to the sum of two not necessarily different prime numbers, or the sum of four not necessarily different prime numbers.

Proof

As a matter of fact, since the G2 Set is equivalent to G1 + G1, every even number greater or equal to 4 can be intended as an element of the G1 Set or an element of the G2 Set, that is to say as the sum of two not necessarily different prime numbers, or the sum of four not necessarily different prime numbers. - QED

Proposition 04

Every element of the E Set can be intended as the sum of two not necessarily different elements of the G1 Set, or, if we prefer, every element of the E Set can be intended as the sum of four prime numbers, not necessarily different from each other.

Proof

As for Corollary 01. 02, every even number greater or equal to 4 is equal to the sum of two not necessarily different prime numbers, or the sum of four not necessarily different prime numbers, and since every element of the E Set is an even number, this implies that every element of the E Set is an element of the G2 Set. - QED

Proposition 05

The E Set is a subset of the G2 Set.

Proof

By reductio ad absurdum, let us suppose that the E Set is not a subset of the G2 Set: if this was the case, then the E set would be made up of three elements, that are the numbers 2, 4 and 6; as a matter of fact, the number 4 and 6 can be intended as $4 = 2 + 2$ and $6 = 3 + 3$, then as elements of the G1 Set, and if 2 was an element of the E Set, this would be coherent with Goldbach's Conjecture, as for Def. A. This implies that no one of these numbers, that is to say 2, 4 and 6, can be considered as an elements of the E Set and then the E Set would be the empty Set. But in this case the E Set wuold be totally contained into the G2 Set, and this contradicts the initial hypothesis, so the E Set is totally contained into the G2 Set. - QED.

Final remarks

This is as far as I went: looking carefully, every possible exception to Goldbach's Conjecture is an almost-exception, because it can be intended as the sum of two not necessarily different elements of the G1 Set, that is to say of two elements which satisfy the conjecture.
The next step would be to understand if the E Set is made up of a finite number of elements or an infinite number of elements: easier said than done, since supposing the E set as infinite does not imply any clear contradiction...