Number notation used

When prime numbers are written as Px, Py, Pz etc., the prime numbers involved are to be understood as not necessarily distinct.
When the prime numbers are written as P1, P2, P3 etc., the prime numbers involved are to be understood as necessarily distinct.
Therefore, a formula such as Nd = Px + (P1 + P2), means that, in the first place, there are always necessarily distinct primes P1 and P2, and that Px is either equal to P1, or equal to P2 or different from P1 and P2.
In what follows, reference will be made to the G1 and G2 Sets as defined in the document: "*On the exceptions to the Goldbach Conjecture*".

Clarification on the conformation of the G2 Set

H. Helfgott, in his Proof of Goldbach's Weak Conjecture, "*The ternary Goldbach conjecture is true*", Corollary 1.1 (to the main theorem), p. 04 of the pdf, states that any number equal to or greater than 4 can be understood as

Npa = Nd - 3

In other words, H. Helfgott considers a generic element of the G2 Set as any odd number to which 3 is subtracted: in this way, the first element of G2 is 4, because 4 = 7 - 3, i.e. 4 = (2 + 2 + 3) - 3, and so on, for all other even numbers.
As seen in "*On the exceptions to Goldbach's Conjecture*", the G2 Set is given by the sum of four not necessarily distinct prime numbers: this means that the approach is a little different.
H. Helfgott obtains the G2 Set as the subtraction of 3 from each odd number expressed as the sum of three not necessarily distinct prime numbers (Npa = (Px + Py + Pz) - 3): it follows, as we said, that the first element of G2 Set is 4.
I obtain the G2 Set as the sum of 3 to each odd number expressed as the sum of three not necessarily distinct prime numbers (Npa = (Px + Py + Pz) + 3): it follows that the first element of the G2 Set would be 10.
Basically, in the formulation given by me, the fundamental skeleton of the G2 Set would be composed of all the odd numbers of Goldbach's weak theorem to which 3 is added: in other words, for every number equal to or greater than 10, there is always at least an element in the G2 Set of the form Npa = (Px + Py + Pz) + 3.
Unfortunately, however, this is still not sufficient to obtain the G2 Set as defined in "*On the exceptions to the Goldbach's Conjecture*", because it has also been shown that the G2 Set is to be understood as G1 + G1: the fundamental skeleton described above cannot guarantee this. The kind reader should think that, among other things, in G2 defined as G1 + G1, the first element is 8 = (2 + 2) + (2 + 2), not 10 as mentioned above, and that elements of the type Npa = 4 P1 are not present, assuming the G2 Set as only composed of elements of the type (Px + Py + Pz) + 3.
This requires explaining how to make ends meet.
By defining the G2 Set as G1 + G1, we continue (hopefully relatively painlessly...) as follows:

1. Take the odd numbers of Goldbach's Weak Theorem and add 3 (Npa = Px + Py + Pz + 3): this is the fundamental skeleton, for every number equal to or greater than 10 there is always at least one element of the G2 Set of the type Npa = (Px + Py + Pz) + 3;
2. On this skeleton, the rest is added: given two not necessarily distinct elements of the Set G1, G (a) and G (b), if added together we have that G (a) + G (b) = Npa, and since we know that Npa = (Px + Py + Pz) + 3, this hooks all the other elements that can be understood as the sum of four not necessarily distinct prime numbers to at least one element of the skeleton, obtaining a G2 Set intended as G1 + G1;

3. Nevertheless, there is a small exception: the number 8 is not intended as Npa = (Px + Py + Pz) + 3, but only as 8 = 4 P1 = (2 + 2) + (2 + 2).

Reductions for even numbers

Proposition 01

Any even number equal to or greater than 8 can be expressed as

- Npa = (P1 + P2) + (P3 + P4);
- Npa = (2 P1) + (P2 + P3);
- Npa = (2 P1) + (2 P2);
- Npa = (2 P1) + (2 P1), i.e. Npa = 4 P1;
- Npa = (2 P1) + (P1 + P2), i.e. Npa = 3 P1 + P2.

Proof

As proved in "*On the exceptions to Goldbach's Conjecture*", the G2 Set is composed by sums of two not necessarily distinct elements of G1.
Therefore, assuming Npa = Px + Py + Pz + Pk, we can have that:

- Px, Py, Pz and Pk are distinct, so we will have Npa = P1 + P2 + P3 + P4;
- Px and Py are not distinct, while Pz and Pk are, so we will have Npa = (2 P1) + (P2 + P3);
- Px and Py are distinct, Pz and Pk are distinct, but Px and Pk are not distinct, so we will have Npa = (2 P1) + (2 P2);
- Px, Py and Pz are not distinct, while Pk is distinct from the other three, so we will have Npa = (2 P1) + (P1 + P2), that is Npa = 3 P1 + P2;
- Px, Py, Pz and Pk are not distinct, so we will have Npa = 2 P1 + 2 P1, i.e. Npa = 4 P1.

- QED

Proposition 02

Every even number of the form Npa = 4 P1 finds at least one equivalent in Npa = (P1 + P2) + (P3 + P4), Npa = (2 P1) + (P2 + P3) or Npa = 3 P1 + P2.
Every even number of the form Npa = (2 P1) + (2 P2) finds at least one equivalent in Npa = (P1 + P2) + (P3 + P4), Npa = (P1 + P2) + (2 P3) or Npa = 3 P1 + P2.

Proof

Assume an even number of the form Npa = 4 P1.
By applying the Bertrand's Postulate between Npa / 2 and Npa, we obtain a prime number, which we name P1'.
By subtracting P1' from Npa, we will get an odd number:

Npa – P1' = Nd

We apply Goldbach's Weak Theorem to Nd, and we will have that

Npa – P1' = Px + Py + Pz
Npa = P1' + Px + Py + Pz

By distinctly expressing Px, Py and Pz we will have:

Npa = (P1' + P1) + (P2 + P3);

$$Npa = (P1' + P2) + (2\ P3);$$
$$Npa = P1' + 3\ P1, \text{ or } Npa = (P1' + P1) + (2\ P1)$$

P1', as obtained with the Postulate applied to Npa, will never be equal to the prime numbers obtained by applying Goldbach's weak theorem to Nd, because it is at least equal to Npa/2, while the first Px, Py and Pz are all less than Npa/2, otherwise the sum P1' + Px + Py + Pz would be greater than Npa.
Therefore, every even number of the form Npa = 4 P1 finds at least one equivalent in Npa = (P1 + P2) + (P3 + P4), Npa = (2 P1) + (P2 + P3) or Npa = 3 P1 + P2.
This proof is also valid for numbers of the type Npa = (2 P1) + (2 P2). Indeed

$$Npa = (2\ P1) + (2\ P2)$$

applying Bertrand's Postulate to Npa, we obtain P1'. We subtract P1' from Npa:

$$Npa – P1' = Nd$$

We apply Goldbach's Weak Theorem to Nd:

$$Npa – P1' = Px + Py + Pz$$
$$Npa = P1' + Px + Py + Pz$$

We express Px, Py and Pz separately, and we will have:

$$Npa = (P1' + P1) + (P2 + P3);$$
$$Npa = (P1' + P1) + (2\ P2);$$
$$Npa = P1' + 3\ P1, \text{ or } Npa = (P1' + P1) + (2\ P1).$$

Here too, P1' and P1 are never the same, for the reasons already expressed above.
Therefore, every even number of the form Npa = (2 P1) + (2 P2) finds at least one equivalent in Npa = (P1 + P2) + (P3 + P4), Npa = (2 P1) + (P2 + P3) or Npa = 3 P1 + P2. - QED

Proposition 03

Any even number equal to or greater than 8 can be expressed as Npa = (Px + Py) + (P1 + P2).
For each element of G2 Set, there is always an element of G1 Set of the type (P1 + P2), as its necessary but not sufficient condition.

Proof

According to Proposition 02, any even number equal to or greater than 8 can be expressed as Npa = (2 P1) + (P2 + P3), Npa = (P1 + P2) + (P3 + P4) or Npa = 3 P1 + P2: as we can see we always have the presence of a number of the type (P1 + P2), from which it can be stated that any even number equal to or greater than 8 can be expressed as Npa = (Px + Py) + (P1 + P2).
Clearly, this means that for each element of G2 Set, there is always at least one element of G1 Set of the type (P1 + P2), as its necessary but not sufficient condition. - QED

Proposition 04

Each even number of the type Npa = 3 P1 + P2 finds a corresponding in Npa = (2 P1) + (P2 + P3) or Npa = (P1 + P2) + (P3 + P4).
If preferred, any even number equal to or greater than 10 can be expressed as Npa = Px + (P1 + P2

+ P3).

Proof

By Proposition 02, we can state that all even numbers can be understood as:

- Npa = 2 P1 + P2 + P3;
- Npa = P1 + P2 + P3 + P4;
- Npa = 3 P1 + P2.

Based on the details on the conformation of G2 Set, reported above, we know that every even number equal to or greater than 10 finds correspondence in at least one element of Set G2 of the form

$$Npa = (Px + Py + Pz) + 3$$

By Proposition 03, there is always at least one element of G1 Set of the type (P1 + P2) as a necessary but not sufficient condition for each element of G2 Set.
Therefore, if every even number Npa equal to or greater than 10 can be interpreted as Npa = (Px + Py + Pz) + 3, and Npa = (Px + Py + Pz) + 3 is an element of the Set G2, and condition necessary but not sufficient for the G2 Set is an element of the G1 Set of the form (P1 + P2), then we can say that

$$Npa = (P1 + P2) + (Px + 3)$$

- QED

Proposition 05 (Property 2 P1)

Between x and 2x, with x equal to or greater than 6, there is always an even number of the form 2 P1 smaller than 2x (Property 2 P1).

Proof

If x is greater than or equal to 2, we would have two cases:

- If x is even, then x = 2h, with h greater than or equal to 1, so Bertrand's Postulate can be applied to the number h, obtaining that there exists a prime number P1 between (h + 1) and (2 h = x). Then the number 2 P1 is included between 2 (h + 1) = 2h + 2 = x + 2, and 2 (2h) = 2x, and therefore also between x + 1 and 2x;
- If x is odd, then x = 2 k + 1, with k greater than or equal to 1. By applying Bertrand's Postulate to the number k, we obtain that there exists a prime number P1 between k + 1 and 2k. Then the number 2 P1 is between 2 (k + 1) = 2k + 2 = x + 1, and 2 (2k) = 2 (x - 1) = 2x - 2, so the number 2 P1 is also between x + 1 and 2x.

As you can see, between x and 2x there is always an even number of the form 2 P1. - QED

Proposition 06

Any even number Npa equal to or greater than 12 of the type Npa = (P1 + P2) + (P3 + P4) can be understood as Npa = 2 P1 + P2 + Cp, where Cp is a coprime number with Npa - 2 P1.

Proof

We assume an even number of the type Npa = P1 + P2 + P3 + P4.
For Property 2 P1, we can always find an even number of type 2 P1' smaller than Npa, which we can subtract:

Npa = P1 + P2 + P3 + P4
Npa - 2 P1 '= Npa'

Applying the Bertrand-Goldbach Theorem (statement and proof can be found here, http://www.dimostriamogoldbach.it/it/strategia-fattatorie/) to Npa we have

Npa - 2 P1 '= Npa'
Npa - 2 P1 '= P2 + Cp
Npa = 2 P1 '+ P2 + Cp

Therefore, every even number Npa = P1 + P2 + P3 + P4 can be understood as Npa = 2 P1' + P2 + Cp, where Cp is a coprime number with Npa - 2 P1. - QED

Reductions for odd numbers

Proposition A

Any odd number equal to or greater than 7 can be expressed as Nd = P1 + P2 + P3, Nd = 2 P1 + P2 or Nd = 3 P1.

Proof

By Goldbach's Weak Theorem, any odd number Nd equal to or greater than 7 can be expressed as Nd = Px + Py + Pk.
If Px, Py and Pk are distinct, we have that Nd = P1 + P2 + P3.
Px and Py are not distinct, but Pk is distinct from the other two we have that Nd = 2 P1 + P2.
If Px, Py and Pk are not necessarily distinct, we have that Nd = 3 P1.
As can be seen, any odd number equal to or greater than 7 can be expressed as Nd = P1 + P2 + P3, Nd = 2 P1 + P2 or Nd = 3 P1. - QED

Proposition B

Every odd number Nd of the type Nd = 3 P1 finds a correspondent in Nd = 2 P1 + P2 or Nd = P1 + P2 + P3. If preferred, any odd number Nd equal to or greater than 7 can be expressed as Nd = Px + (P1 + P2).

Proof

By Proposition 04, we can state that any even number Npa equal to or greater than 10 can be written as Npa = Px + (P1 + P2 + 3). And therefore

$$Npa = Px + (P1 + P2 + 3)$$
$$Npa = (Px + P1 + P2) + 3$$
$$Npa - 3 = (Px + P1 + P2)$$

By distinctly expressing Px, we will have

$$(2\ P1 + P2) \text{ or } (P1 + P2 + P3)$$

Given the link between the G2 Set and Goldbach's weak theorem, Npa - 3 guarantees that there are all odd numbers and none are missing (for example: 10 - 3 gives us 7; 12 - 3 gives us 9; 14 - 3 gives us 11 and so on: as we can see, there are clearly all odd numbers equal to or greater than 7, i.e. all numbers covered by Goldbach's Weak Theorem). - QED

Final remarks

Why get stuck on such matters? There are good reasons for doing this.
The main objective is to prove what is known as the "Lemoine-Levy conjecture", according to which, for every odd number Nd equal to or greater than 7 we have that Nd = 2 Px + Py. To tell the truth, more specifically, the intention would be to demonstrate a slightly stronger formulation of this Conjecture, the one for which Nd = 2 P1 + P2.

1. Keep in mind that, if Nd = 2 Px + Py, then Npa = 2 Px + Py + Pz: this helps to standardize the G2 Set, and to find a formulation which is also uniform for any exceptions to the Goldbach's Conjecture;
2. Keep in mind that, if Nd = 2 P1 + P2, then we can affirm that Npa = P1 + (2 P2 + 1), which is a more powerful affirmation than the previous one above, but above all it is even more powerful of Chen's Theorem.

This should make it clear why people get so stuck on such issues.